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Can quantization improve error performance in CDMA?

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Abstract

A K-user direct-sequence spread-spectrum code-division multiple-access (CDMA) system with $(q \ll \log_2 K)$-bit baseband signal quantization at the demodulator is considered. It is shown that additionally quantizing the $K+1$ level output signal of the CDMA modulator into q bits improves significantly the average bit-error performance in a non-negligible regime of noise variance, σ^2, and user load, β, under various system settings, like additive white Gaussian noise (AWGN), Rayleigh fading, single-user detection, multi-user detection, random and orthogonal spreading codes. For the case of single-user detection in random spreading AWGN-CDMA, this regime is identified explicitly as $\sigma < \gamma(q)\sqrt{\beta}$, where $\gamma(q)$ is a certain pre-factor depending on q, and the associated BER improvement is derived analytically for $q = 1, 2$. For the other examined system settings, computer simulations are provided, corroborating this interesting behavior.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The detrimental effect of quantization, i.e., the process of reducing the number of bits representing a signal, is a common conception in digital signal processing [1]. In this paper, we show that signal quantization can actually have a constructive, rather than destructive, role in improving the end-to-end error performance of a transmission system.

To this end, a direct-sequence spread-spectrum code-division multiple-access (DS/SS-CDMA) system is examined [2]. The bit-error rate (BER) performance loss due to quantization performed on the baseband analog signal at the input of a conventional CDMA demodulator (i.e., matched filter) was thoroughly investigated in several contributions [3–8].

On top of this practical system model, we show that quantizing also the SS signal at the output of the modulator, prior to transmission, can yield, for a certain non-negligible regime of noise level and user load, a substantial improvement in BER performance w.r.t. the standard transceiver. Simulation results are also provided, corroborating the analysis.

The paper is organized as follows. Section 2 introduces the dual-transceiver quantization scheme. Section 3 discusses the error performance of the proposed quantization scheme in comparison with the conventional receiver-only quantization, providing both analytical and simulation results for common CDMA system settings. We conclude in section 4.

2. Transceiver quantization

Consider a K-user, coherent, synchronous binary-phase shift-keying (BPSK) signaling DS/SS-CDMA system's downlink employing binary spreading codes of N chips over an additive white Gaussian noise (AWGN) and (frequency nonselective, slowly) fading channel. The q-bit quantized baseband sample of the received signal of such a system can be described by

$$y_\mu^r = \Delta^r\{\delta y_\mu^t + n_\mu\} = \Delta^r\left\{\frac{\delta}{\sqrt{N}}\sum_{k=1}^{K} s_{\mu k} b_k + n_\mu\right\}, \tag{1}$$

where $s_{\mu k} = \pm 1$ ($\mu = 1, \ldots, N, k = 1, \ldots, K$) are the binary spreading chips. The deterministic chip waveform is assumed to be of unit energy; b_k is the information source binary symbol transmitted by the kth user and is taken from a Bernoulli 1/2 process; n_μ is an AWGN sample taken from the Gaussian distribution $\mathcal{N}(0, \sigma^2)$; The scalar δ represents the envelope of the equivalent lowpass fading channel. The $\Delta^r\{\cdot\}$ function denotes the quantization operation at the receiver, clipping nonlinearly the summation of the faded transmitted signal, δy_μ^t, and the noise into a q-bit finite word. We assume a perfect power-control mechanism yielding unit energy transmissions (per user per symbol). An extension to quadrature phase-shift keying (QPSK) modulation is straightforward.

This model (1), as depicted in figure 1(a) for $\delta = 1$, is a customary description of the received signal at the input of a commercial CDMA receiver. The following modification is proposed. Let the transmitted SS signal, y_μ^t, which consists of $K + 1$ discrete amplitude levels, be also quantized (using a clipping function $\Delta^t\{\cdot\}$) at the output of the transmitter into a reduced description of $q \ll \log_2 K$ bits. Thus, the quantized received sample gets the new form

$$y_\mu^r = \Delta^r\{\delta\Delta^t\{y_\mu^t\} + n_\mu\} = \Delta^r\left\{\delta\Delta^t\left\{\frac{1}{\sqrt{N}}\sum_{k=1}^{K} s_{\mu k} b_k\right\} + n_\mu\right\}. \tag{2}$$

Applying the matched filter on these samples (2), one obtains the soft output

$$\psi_k = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} s_{\mu k}\Delta^r\left\{\delta\Delta^t\left\{\frac{1}{\sqrt{N}}\sum_{k'=1}^{K} s_{\mu k'} b_{k'}\right\} + n_\mu\right\}. \tag{3}$$

In the following section, it is shown that such a dual-quantization scheme (figure 1(b)) on both ends of the transceiver renders a significant BER improvement for a non-negligible range of system parameters. To do so, an asymptotic large-system analysis is performed, assuming N and K are large, yet the user load factor $\beta \triangleq K/N$ is kept finite. However, we would like to emphasize that the experimental study substantiates the validity of our analysis results for a finite number of users, K, and a practical spreading factor, N, as well.

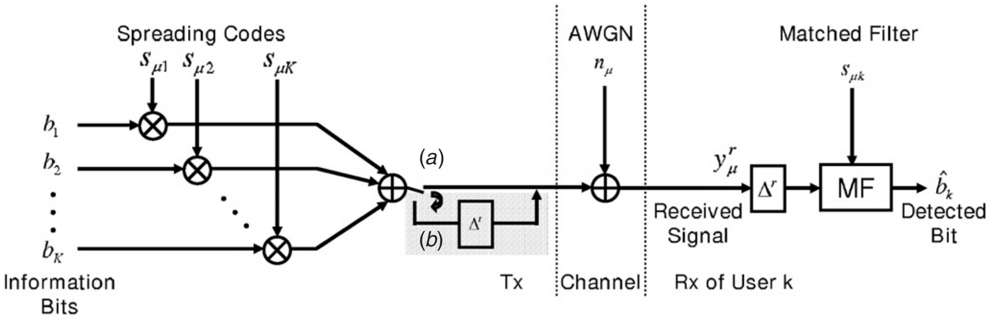

Figure 1. (*a*) Conventional BPSK-CDMA transmission over an AWGN channel with matched filter input quantization at the receiver. (*b*) Adding transmitter quantization (shaded block).

In addition, in order to perform the analysis explicitly, we examine the cases of 1 and 2-bit quantization (i.e., $q = 1, 2$). Thus, for the case of 2-bit description the transceiver's clipping functions are defined by

$$\Delta^t\{x\} \triangleq \eta \cdot \text{sgn}\{x\} \cdot \begin{cases} A & \text{if} \quad |x/\sqrt{\beta}| < a \\ B & \text{if} \quad |x/\sqrt{\beta}| \geqslant a \end{cases} \tag{4}$$

and

$$\Delta^r\{x\} \triangleq \text{sgn}\{x\} \cdot \begin{cases} E & \text{if} \quad |x| < e \\ F & \text{if} \quad |x| \geqslant e, \end{cases} \tag{5}$$

where $\text{sgn}\{\cdot\}$ denotes the hard-decision signum function. The quantized amplitudes A, B, E, F are free scalars to be determined via a BER optimization, while the thresholds are arbitrarily chosen as $a \triangleq (B - A)/2$ and $e \triangleq (F - E)/2$.

What is the role of the pre-factor η in $\Delta^t\{x\}$ (4) and how is it being determined? In the large-system limit, the average power of the (un-quantized) transmitted SS signal sample, y^t_μ, is equal to β (i.e., the sum-power of K-user chips, each with power $1/N$). In the quantized case (4), the equivalent transmitted power is $\eta^2(2P_AA^2 + 2P_BB^2)$ with $P_A \triangleq 1/\sqrt{2\pi}\int_0^a \exp(-x^2/2)\,\mathrm{d}x$ and $P_B \triangleq 1/\sqrt{2\pi}\int_a^\infty \exp(-x^2/2)\,\mathrm{d}x$. For fairness of comparison, equating the power consumption of both systems, the pre-factor, η, is easily determined,

$$\eta = \sqrt{\frac{\beta}{2P_AA^2 + 2P_BB^2}}. \tag{6}$$

Prior to addressing its error performance, we would like to emphasize that the implementation of the proposed quantization technique is more natural in the context of CDMA downlinks (from the fixed base station to mobile users), where joint processing (e.g., the joint quantization of the superposed signal) at the transmitter can (and should) be applied. Another popular example of joint processing at the transmitter is transmitter pre-coding for combating multiple-access interference in CDMA downlinks [9]. Nevertheless, the proposed method may also be relevant for the uplink (from mobile user to fixed base station) of certain practical CDMA systems, like the leading third-generation cellular standardization (UMTS, [10]). In UMTS each mobile user is assigned with several spreading codes serving him in the simultaneous transmission of different types of information streams (e.g., voice, data, video). Hence, in this case joint processing is conceptually feasible also on the user level, thus relevant even for such uplink.

3. Error performance: analysis, results and discussion

3.1. Random spreading codes

First, let us consider the popular case where the (long) spreading codes are modeled by an N-length random sequence of binary spreading chips, $s_{\mu k} = \pm 1$, being chosen independently and equiprobably.

3.1.1. Single-user detection. Applying a single-user matched filter (SUMF) for the CDMA detection problem results in binary decisions

$$\hat{b}_k = \text{sgn}(\psi_k). \tag{7}$$

AWGN. In this section, we assume a non-fading channel, i.e., $\delta = 1$. For the case of $q = 2$, a straightforward probabilistic analysis of the average error performance for the standard receiver-only quantization in the large-system limit gives the BER,

$$\Pr(\hat{b}_k \neq b_k)^{r}_{q=2} = (1 - \text{erf}(\alpha(E,F)/\sqrt{\beta + \sigma^2}))/2, \tag{8}$$

where

$$\alpha(E,F) \triangleq \sqrt{\frac{1}{2\pi(P_E E^2 + P_F F^2)}}(E + (F - E)\exp(-e^2/2)) \tag{9}$$

with

$$P_E \triangleq \tfrac{1}{2}\,\text{erf}(e/\sqrt{2}), \tag{10}$$

$$P_F \triangleq \tfrac{1}{2}(1 - \text{erf}(e/\sqrt{2})) \tag{11}$$

and $\text{erf}(x) \triangleq 2/\sqrt{\pi}\int_0^x \exp(-t^2)\,\mathrm{d}t$ denoting the error function.

The exact values of the amplitudes E and F, found to be independent of β and σ^2, are determined by minimizing $\Pr(\hat{b}_k \neq b_k)^{r}_{q=2}$. Using numerical optimization, one finds the optimal amplitude values $E^* \simeq 0.84$ and $F^* \simeq 2.8$ for the quantizer at the receiver, yielding $\alpha^* \simeq 0.66$.

A similar analysis can be performed for the proposed dual-quantization scheme resulting in

$$\Pr(\hat{b}_k \neq b_k)^{t+r}_{q=2} = (1 - \text{erf}(\alpha(E,F,\beta,\sigma^2)))/2, \tag{12}$$

where

$$\alpha(E,F,\beta,\sigma^2) \triangleq \frac{\sqrt{\beta}\sum_{i=\{\pm E,\pm F\}} i \sum_{j=\{\pm A,\pm B\}} P(i|j)P(j)}{\sqrt{2\sum_{i=\{\pm E,\pm F\}} i^2 \sum_{j=\{\pm A,\pm B\}} P(i|j)P(j)}} \tag{13}$$

and

$$P(i|j) \triangleq \frac{1}{\sqrt{2\pi\sigma^2}}\int_l^u \exp(-x^2/(2\sigma^2))\,\mathrm{d}x, \tag{14}$$

where l and u denote the 16 different combinations of lower and upper integration limits, respectively, depending on the values of the i and j terms (e.g., for $i = +E$, $j = +A$ one gets $l = -A\eta$, $u = -A\eta + e$).

In order to simplify the numerical optimization, the optimized amplitude values of A and B at the transmitter, for this dual-quantization scheme, were taken from the previous receiver-only quantization case. Since a receiver-only quantization can be shown to be equivalent,

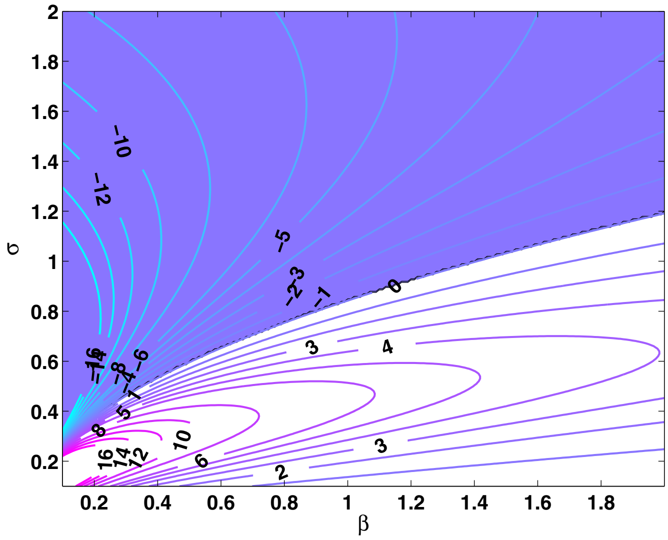

Figure 2. The difference (in %) in BER between the proposed dual-transceiver quantization and standard receiver-only quantization schemes in random spreading AWGN-CDMA with SUMF detection for $q = 1$-bit quantization. Improvement is obtained for β and σ values within the unshaded area. Contours denote equal difference lines in the β–σ plane.

as far as error performance concerns, to a transmitter-only quantization, one can substitute $A^* = E^*, B^* = F^*$ for the quantization levels at the transmitter, using the previously calculated E^* and F^* derived for the receiver-only quantization scheme. This leaves us with the computation of the receiver quantization's BER-minimizing amplitude values E^* and F^* (which are themselves a function of β and σ^2).

Evidently, in the case of 1-bit quantization the error performance for the standard receiver-only quantization, $\Pr(\hat{b}_k \neq b_k)^r_{q=1}$, and novel dual-quantization schemes, $\Pr(\hat{b}_k \neq b_k)^{t+r}_{q=1}$, is encapsulated within the above analysis by using hard-decision signum functions, i.e. substituting $E = F = 1$ and $A = B = E = F = 1, \eta = \beta$, respectively. Explicitly, we get

$$\Pr(\hat{b}_k \neq b_k)^r_{q=1} = \frac{1}{2}\left(1 - \mathrm{erf}\left(\sqrt{\frac{1}{\pi(\beta+\sigma^2)}}\right)\right), \tag{15}$$

$$\Pr(\hat{b}_k \neq b_k)^{t+r}_{q=1} = \frac{1}{2}\left(1 - \mathrm{erf}\left(\mathrm{erf}\left(\sqrt{\frac{\beta}{2\sigma^2}}\right)\sqrt{\frac{1}{\pi\beta}}\right)\right). \tag{16}$$

Figures 2 and 3 compare the BER performance of the proposed scheme (figure 1(*b*)) to that of the standard scheme (figure 1(*a*)) for 1-bit and 2-bit quantization, respectively, in the case of a random spreading AWGN-CDMA channel and SUMF detection. Labeled contours of equal BER differences (in percentage) between the schemes, i.e.,

$$\frac{\Pr(\hat{b}_k \neq b_k)^{t+r}_q - \Pr(\hat{b}_k \neq b_k)^r_q}{\Pr(\hat{b}_k \neq b_k)^r_q} \cdot 100 = \mathrm{const}, \tag{17}$$

are drawn as a function of the user load β and the noise standard deviation σ. Clearly, two regimes can be identified. First, the regime (shaded) of β and σ in which quantizing the

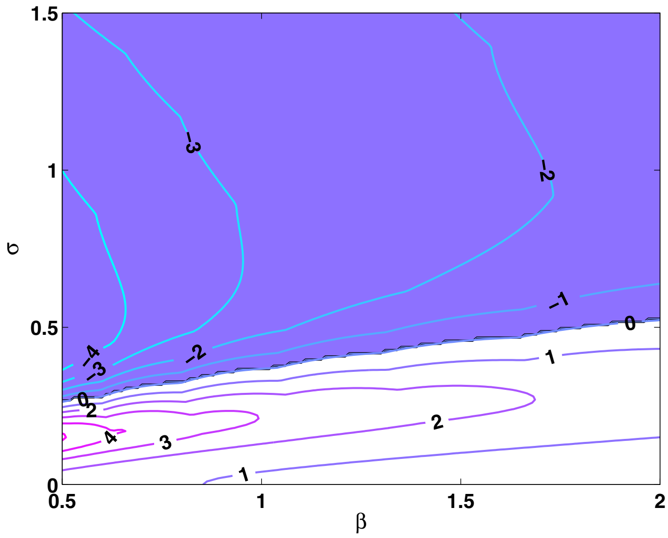

Figure 3. BER difference for random spreading AWGN-CDMA with SUMF detection and $q=$ 2-bit quantization.

transmitted SS signal deteriorates error performance. Interestingly, there is a second non-negligible regime (unshaded) of pairs of β and σ where a significant improvement in BER is observed due to the proposed additional transmitter quantization (of up to approximately 16% for $q = 1$ and 4% for $q = 2$, in the plotted axes range). The threshold between the two regimes (i.e., the 0% contour) scales, asymptotically, according to $\sigma \simeq \gamma(q)\sqrt{\beta}$, with the pre-factors $\gamma(q = 1) = 0.87$ and $\gamma(q = 2) = 0.35$. The size of the unshaded regime decreases as we increase the number of description bits q.

The analytical results (8)–(16) were corroborated via large-system (e.g., taking $N = 500$ and $K = 1000$ for the $\beta = 2$ case) simulations producing results in perfect agreement with the analysis and figures 2 and 3. We would like to emphasize that a similar error performance improvement behavior was simulated for relatively small (e.g., taking $N = 5$ and $K = 10$ for the $\beta = 2$ case) CDMA systems, thus validating the attractiveness of the proposed scheme for CDMA systems of practical size.

The explanation for this potential improvement in error performance over standard CDMA transmission arises from the 'pre-processing' mechanism obtained by quantizing the transmitted signal. Let us describe this 'pre-processing' mechanism using an example. Consider the case of $q = 1$ bit and $K \to \infty$. Under standard receiver-only quantization the probability density function of the received signal, prior to quantization, can be described by a Gaussian, $\mathcal{N}(0, \beta + \sigma^2)$ (figure 4(*a*)), while for the proposed scheme the un-quantized received signal is described by two Gaussians, $\mathcal{N}(\Delta^t, \sigma^2)$ (figures 4(*b*) and (*c*)). In both cases the probability distribution of the quantized received signal can be described by two delta functions (depicted as red line at ± 1 in all three figures). Let us examine the two extreme cases of AWGN. Starting with the low noise regime, as drawn in figure 4(*b*), it can be seen that for the proposed dual-transceiver quantization the effect of small AWGN is negligible, hence the probability of getting the wrong value of a certain received signal is very small, validating the attractiveness of the 'pre-processing' mechanism in this regime.

Next, for the case of high AWGN, it is clear from figure 4(*c*) that for dual quantization there is a finite probability to obtain a wrong quantized received signal with respect to the

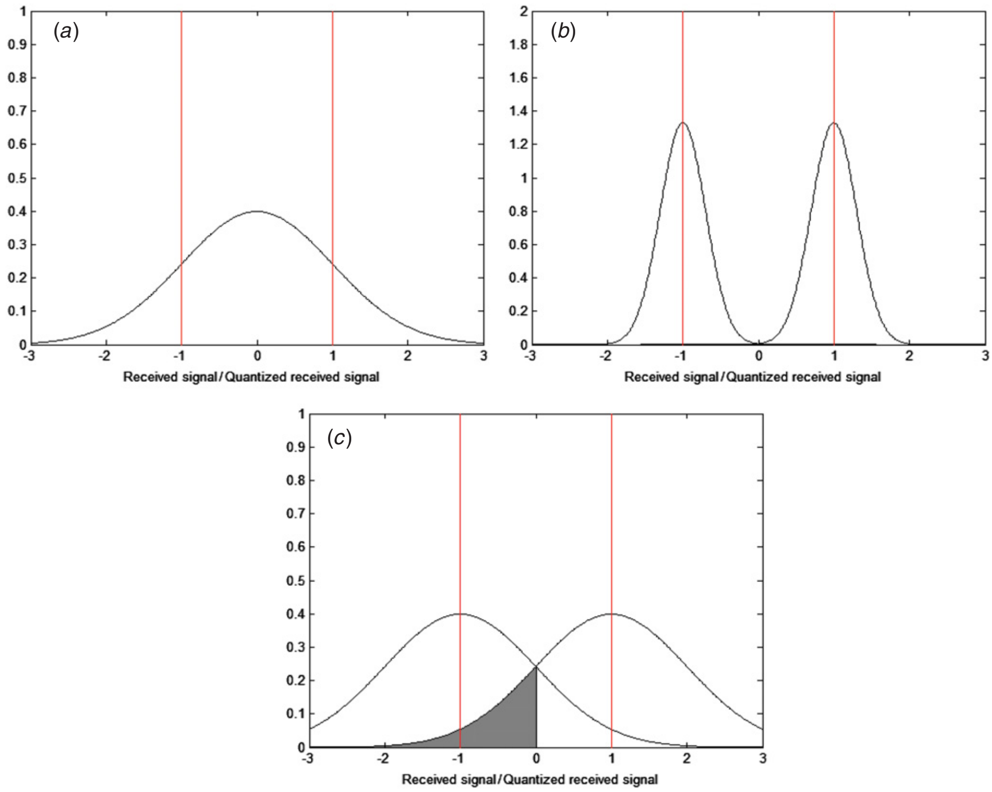

Figure 4. (*a*) Representation of a certain received signal, y_μ^r, when using the standard receiver-only quantization. (*b*) Representation of a certain received signal when using the proposed dual-transceiver quantization, in case where the variance of the AWGN channel, σ^2, is low. (*c*) Representation of a certain received signal, while using dual-transceiver quantization, when the variance of the AWGN channel, σ^2, is high. In all three graphs the red lines represent the received signal after quantization in the case of $q = 1$.

quantized transmitted signal. For example, the case where the quantized transmitted signal is $y_\mu^t = 1$, while the received signal is $y_\mu^r < 0$, is depicted as a gray area in figure 4(*c*).

Focusing on the matched-filter equation (3), it is easy to see that it requires $\mathcal{O}(N)$ randomly selected wrong quantized received signals in order to erroneously detect a certain wrong transmitted bit. Such a case occurs only if the gray area, depicted in figure 4(*c*), has a finite probability. This probability increases as a function of the noise variance. As a result, for a given β and large enough σ^2, a deterioration in the BER performance is obtained under dual quantization.

In the remainder of this paper, we address the error performance of dual-transceiver quantization under the system setup, other than random spreading AWGN-SUMF, including fading, multi-user detection (MUD) and orthogonal spreading codes. The following error performance results were obtained using computer simulations, and again similar behavior emerges for both large and relatively small systems.

Fading. Figure 5 shows the BER difference contours over the β–σ plane, based on simulations, for 1-bit quantization in a similar setting to the one used to produce figure 2, except that here a frequency nonselective, slowly fading channel with coherent demodulation is assumed. Hence,

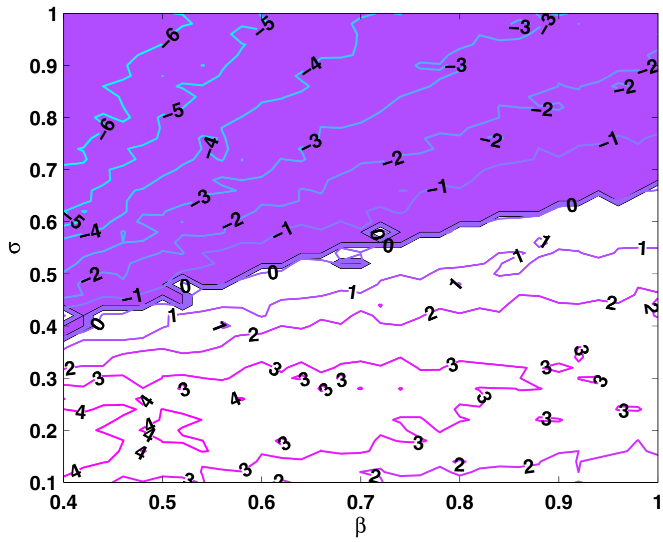

Figure 5. BER difference for random spreading Rayleigh fading CDMA with SUMF detection and $q = 1$-bit quantization.

the scalar δ is taken from a Rayleigh distribution (with $\Omega = 1$). Again, a BER improvement regime is found due to the additional quantization at the transmitter.

3.1.2. Multi-user detection. In order to examine the performance of the proposed dual-transceiver quantization scheme under MUD, we have simulated a tractable iterative CDMA multi-user detector, recently introduced by Kabashima [11], which is based on the celebrated belief propagation algorithm (BP, [12, 13]). This novel BP-based MUD algorithm exhibits considerably faster convergence than conventional multistage detection [14] without increasing computational cost significantly. It is considered to provide a nearly-optimal detection when the spreading factor N is large and the noise level is known.

Similarly to multistage detection, at each iteration cycle t this detector computes tentative soft decisions η_k^t for each user transmission, of the form

$$\eta_k^t = \tanh\left(h_k^t\right). \tag{18}$$

The parameters η_k^t and h_k^t (h_k^0 is initialized by equating it to the SUMF output, ψ_k (3)) are coupled and being iteratively computed using the following recipe:

$$U_k^t = A^t \sum_{l=1}^{K} W_{kl}\eta_l^t + A^t\beta(1 - Q^t)U_k^{t-1}, \tag{19}$$

$$h_k^{t+1} = R^t h_k^0 - U_k^t + A^t\eta_k^t(1 - Q^t)U_\mu^{t-1}, \tag{20}$$

$$R^t = A^t + A^t\beta(1 - Q^t)R^{t-1}, \tag{21}$$

where $W_{kl} \triangleq \sum_{\mu=1}^{N} s_{\mu k}s_{\mu l}/N$, $Q^t \triangleq \sum_{k=1}^{K}\left(\eta_k^t\right)^2/K$, $A^t \triangleq (\sigma^2 + \beta(1 - Q^t))^{-1}$ and tentative hard-decisions are taken by $\hat{b}_k^t = \mathrm{sgn}\left(\eta_k^t\right)$. Producing $\hat{b}_k^t \equiv \hat{b}_k^{t+1}, \forall k$, serves as the convergence criterion.

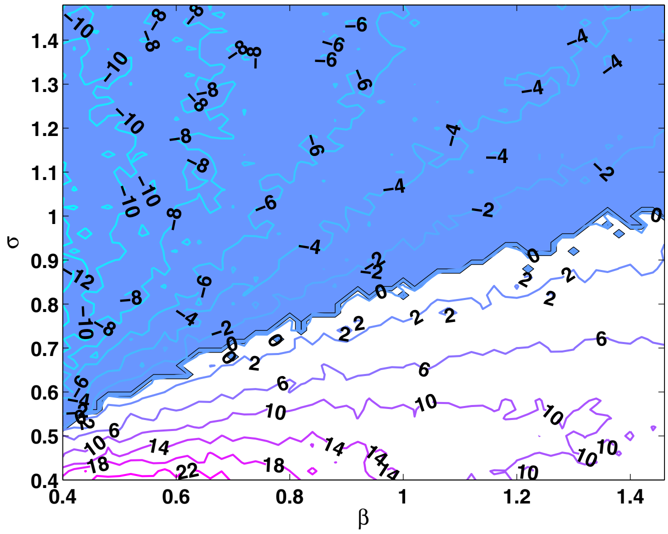

Figure 6. BER difference for random spreading AWGN-CDMA with MUD detection and $q = 1$-bit quantization.

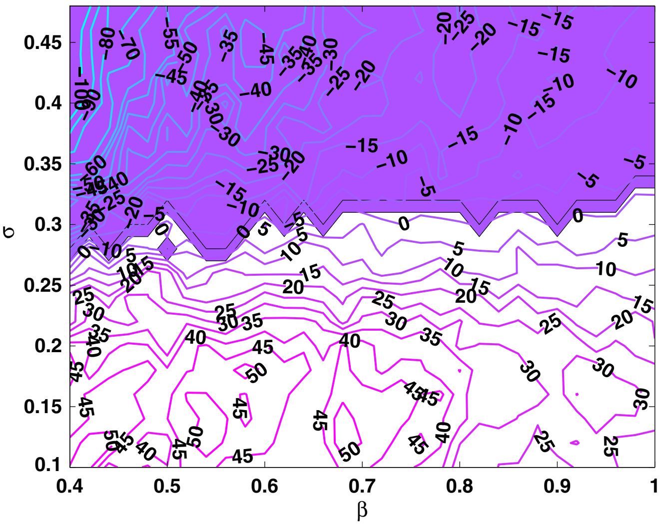

Figure 7. BER difference for random spreading AWGN-CDMA with MUD detection and $q = 2$-bit quantization.

AWGN. Figures 6 and 7 display the BER difference as a function of β and σ, for 1-bit and 2-bit quantization, respectively, in the case of a random spreading AWGN-CDMA channel and Kabashima's tractable MUD [11]. Significant error performance improvement is observed for this case as well. In the plotted range, a 22% for $q = 1$ and 50% for $q = 2$ improvement is observed.

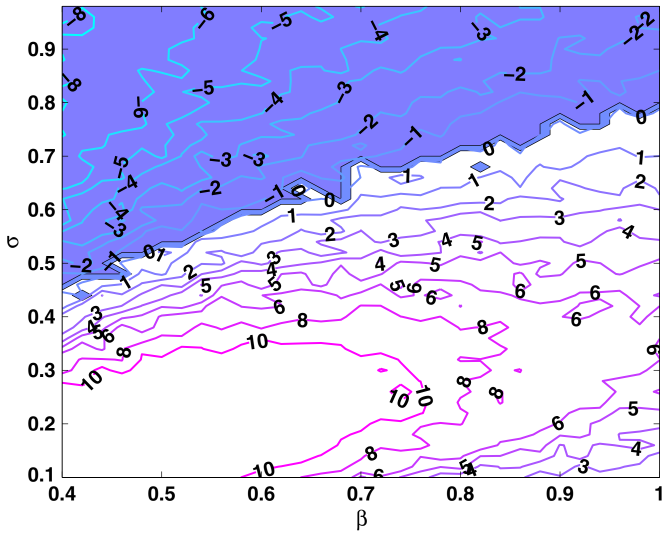

Figure 8. BER difference for random spreading Rayleigh fading CDMA with MUD detection and $q = 1$-bit quantization.

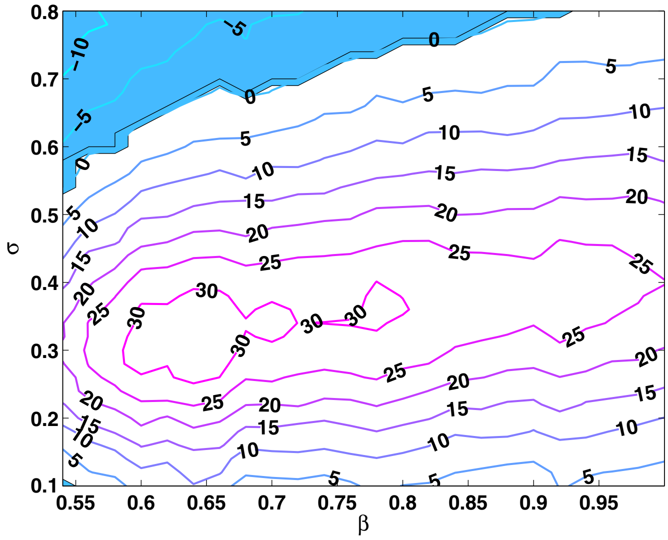

Figure 9. BER difference for AWGN orthogonal CDMA with SUMF detection and $q = 1$-bit quantization.

Fading. Figure 8 demonstrates the BER equal-difference lines over the β–σ plane for 1-bit quantization, this time under flat Rayleigh fading (with $\Omega = 1$). In the figure, up to 10% improvement contours due to the additional quantization at the transmitter are presented.

3.2. Orthogonal spreading codes

We have repeated our error performance simulations for the case of orthogonal, rather than random, spreading codes (constructed from Hadamard matrices).

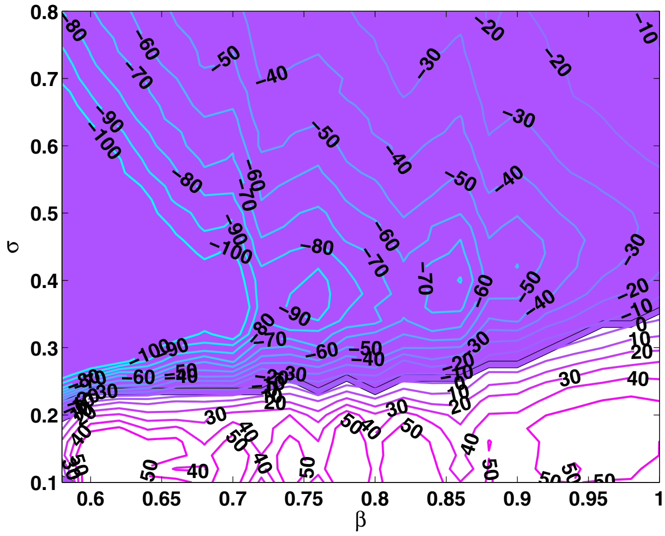

Figure 10. BER difference for AWGN orthogonal CDMA with SUMF detection and $q = 2$-bit quantization.

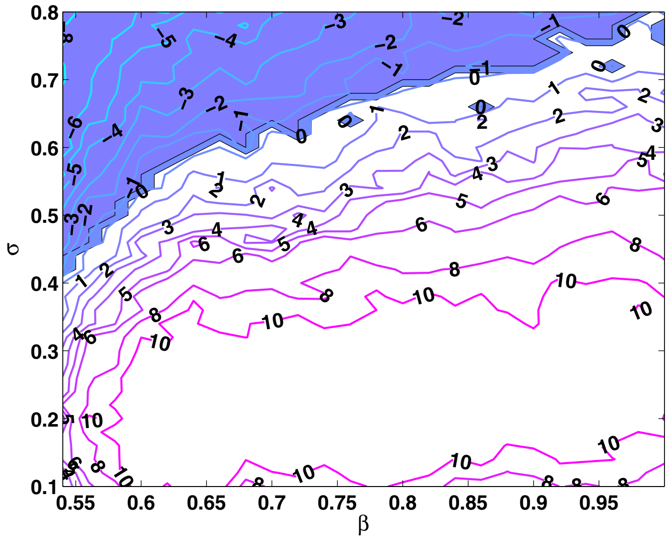

Figure 11. BER difference for Rayleigh fading orthogonal CDMA with SUMF detection and $q = 1$-bit quantization.

3.2.1. Single-user detection. Since orthogonal spreading is used, the correlation matrix equals to the identity matrix and the (Kabashima) multi-user detector reduces back to the simple SUMF. Thus, only single-user detection is examined in this scenario.

AWGN. Figures 9 and 10 illustrate the BER difference contours over the β–σ plane, for 1-bit and 2-bit quantization, respectively, in the case of an orthogonal spreading AWGN-CDMA channel and SUMF detection. Improvement contours of up to 30% for $q = 1$ and 50% for $q = 2$ are drawn.

Fading. Figure 11 presents the BER difference over the β–σ plane, based on simulations, for 1-bit quantization in a similar setting, except that here again the scalar δ is taken from a Rayleigh distribution ($\Omega = 1$).

Once again we would like to emphasize that also for the case of orthogonal spreading codes, a similar error performance improvement behavior was simulated for relatively small (e.g., $N = 5$, $K = 10$ for $\beta = 2$ case) CDMA systems, thus validating the attractiveness of the proposed scheme for CDMA systems of practical size.

4. Conclusion

A CDMA transceiver scheme introducing dual quantization is proposed. A (counter-intuitive) improvement in error performance is either analytically derived or empirically observed w.r.t. standard CDMA transmission under various system models. It is well-known [3, 4] that a practical receiver-only quantization system with $q = 4$ is enough to approach the error performance of a CDMA system without any quantization. Thus, in addition to its theoretical importance in revealing the positive role of the notion of quantization, the proposed dual-transceiver quantization scheme is of major interest in the case of small q, potentially improving the error performance of low-complexity CDMA transceivers.

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